

# An Approach For Concept Drift Detection in a Graph Stream Using a Discriminative Subgraph

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## Abstract

- With the emergence of the complex network (social media, sensor network, world wide web) the interest in graph mining has increased.
- In a streaming scenario, the concept to be learned might change over time.
- We propose a three-step approach to detect concept-drift detection on the graph streams:-
  - Subgraph Generation:** - Find subgraphs (discriminative) for graphs in the stream.
  - Entropy Calculation:** - Measures distribution of current window in the graph stream by computing the entropy.
  - Drift Detection:** - Detect drift in the series of entropy values by moving one step forward in the sliding window.

## Research Objective

Design a state-of-the-art concept-drift detection method on graph streams.

## Methodology

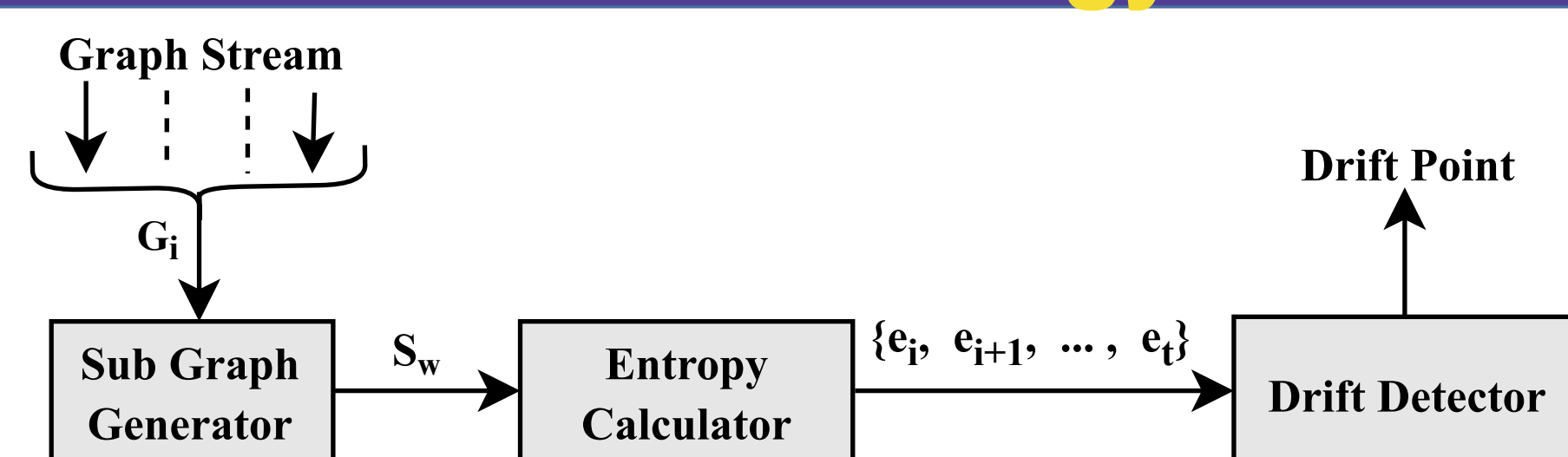


Fig 1. Methodology for Concept Drift Detection

## Algorithm

**Algorithm 1:** Discriminative Subgraph-based Drift Detector (DSDD)

```

Input: Window Size = W
Graph Stream  $G_s = \{G_1, G_2, \dots, G_t, \dots\}$ 
Output: Drift Points, False Alarm
1 Set  $t = 0, n = 50, k = 10, \alpha = 0.1, fold = 5$ 
2 Initialize
3  $S_w = \{\}$ ; Holds subgraph and their count in  $G_t$ 
4  $Drift = \{\}$ 
5  $t_{buffer} \leftarrow (2n + k - 1)$ 
6  $i_{next} \leftarrow (2n + k - 1)$ 
7 while not end of stream do
8    $t \leftarrow t + 1$ 
9    $S_t \leftarrow getDiscriminativeSubgraphs(G_t)$ 
10   $S_w \leftarrow S_w \cup S_t$  //update  $S_w$  with new subgraphs
11  if  $t \geq W$  then
12     $i \leftarrow t - W$ 
13     $e[i] \leftarrow getWindowEntropy(S_w)$  //using Eq. 2
14    if  $i \geq t_{buffer}$  and  $i == i_{next}$  then
15       $e_{cur} \leftarrow e[i - t_{buffer} : i]$ 
16       $\hat{PE}_\alpha \leftarrow getChangeScore(e_{cur}, n, k, \alpha, fold)$ 
17      if  $\hat{PE}_\alpha > \eta$  then
18        Append  $t$  in Drift
19         $i_{next} \leftarrow i + (2n + k)$ 
20      else
21         $i_{next} \leftarrow i + 1$ 
22      end
23    end if
24     $S_w \leftarrow S_w - S_{t-W}$ 
25  end if
26 end
    
```

## Step 1– Subgraph Generation

- Find discriminative subgraphs using the minimum description length (MDL) principle.
- Discriminative subgraph is the one that minimizes :
 
$$M(S, G) = DL(G|S) + DL(S)$$
 where G is the entire graph, S is the subgraph, DL(S) is the description length of the subgraph, DL(G|S) is the description length of G after compressing it using S.

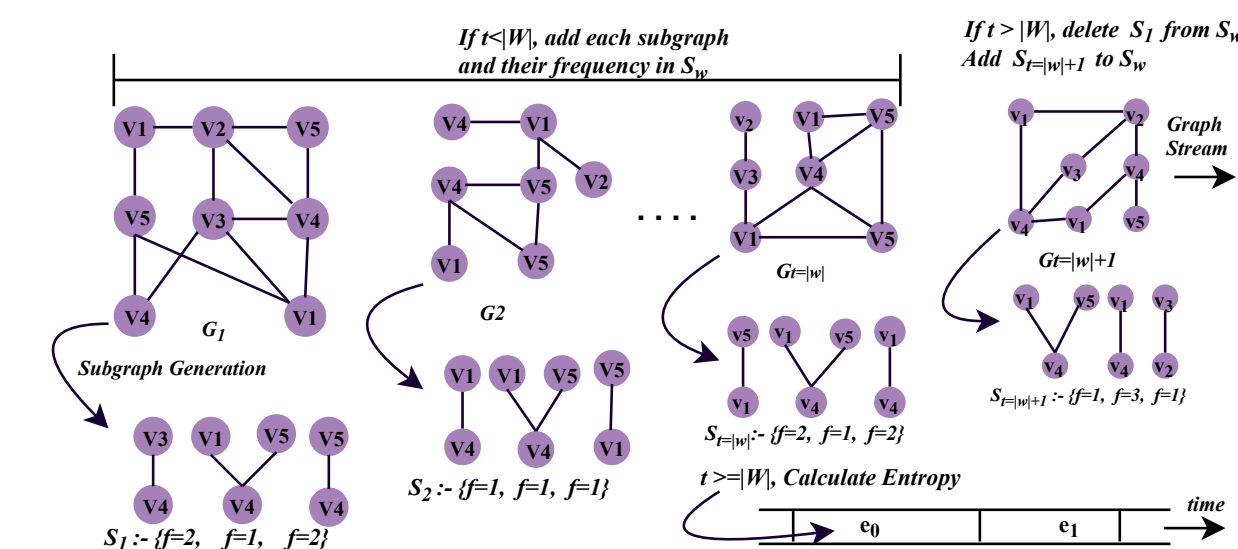


Fig 2. Illustration of discriminative subgraph generation from graph stream

## Step 2 – Entropy Calculation

- The probability of each discriminative subgraph  $S_i$  in the current window with respect to the graph  $G_i$  is:

$$P(S_i|G_i) = \frac{r_{S_i}^{G_i}}{\sum_{j=1}^{|G|} r_{S_j}^{G_j}} \quad (1)$$

- Entropy of the window based on subgraphs w.r.t. the graphs in  $W$  is defined as:

$$e(W) = - \sum_{j=1}^{|S|} P(S_j) \sum_{i=1}^N P(S_i|G_i) \log_2 P(S_i|G_i) \quad (2)$$

where  $P(S_i)$  is the fraction of subgraphs  $S_i$  in  $W$

$$P(S_i) = \frac{S_i}{\sum_{j=1}^N S_j}, \quad N \text{ is the total number of subgraph in } W$$

## Dataset and the Graph Structure

Graph Stream	# of Graph in		Drift Points
	Synthetic	Real World	
SD1	1000	1000	1001
SD2	3000	3000	every 1001 <sup>st</sup> step
	DBLP	1000	1000
	AIDS	1600	400
	Muta	2401	1936
	DoS Attack	2979	221

Table 1. Total graph in graph stream dataset and the drift point

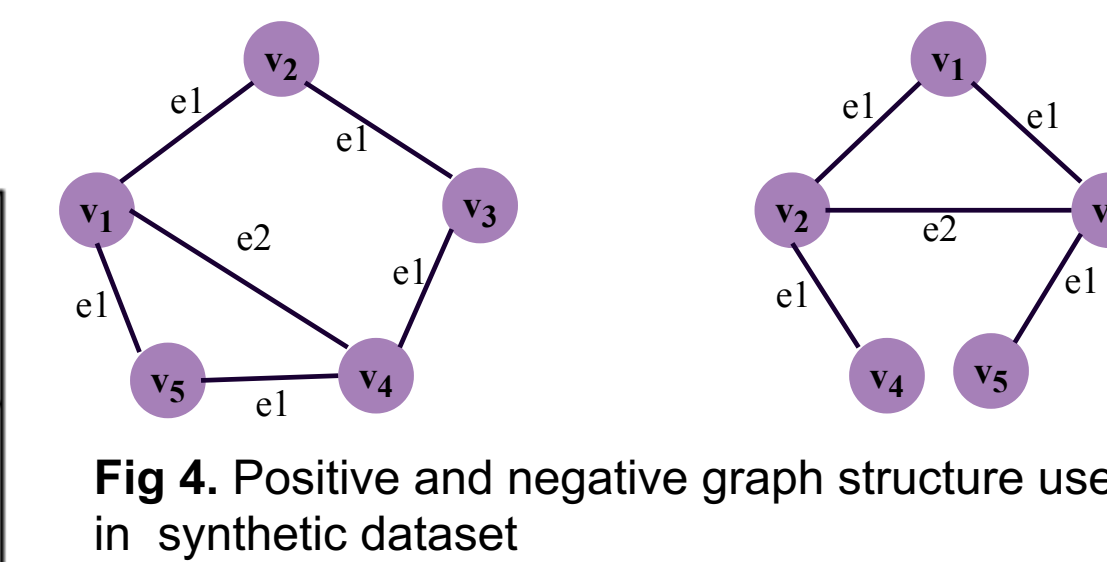


Fig 4. Positive and negative graph structure used in synthetic dataset

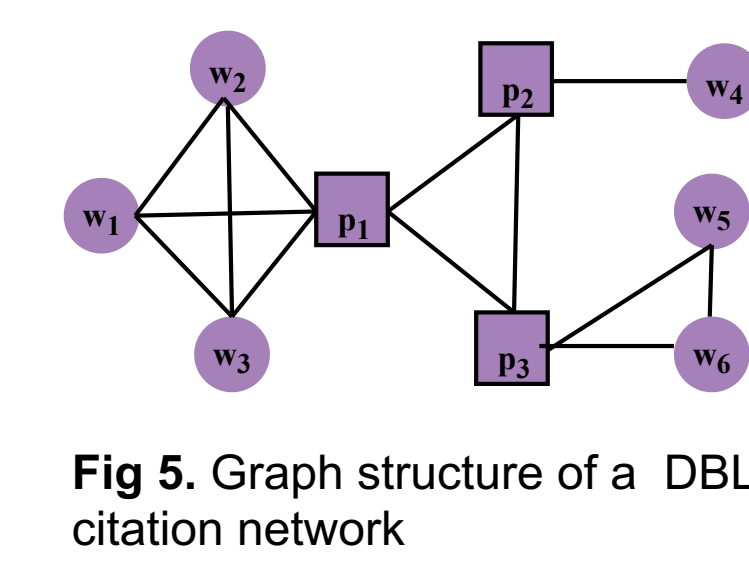


Fig 5. Graph structure of a DBLP citation network

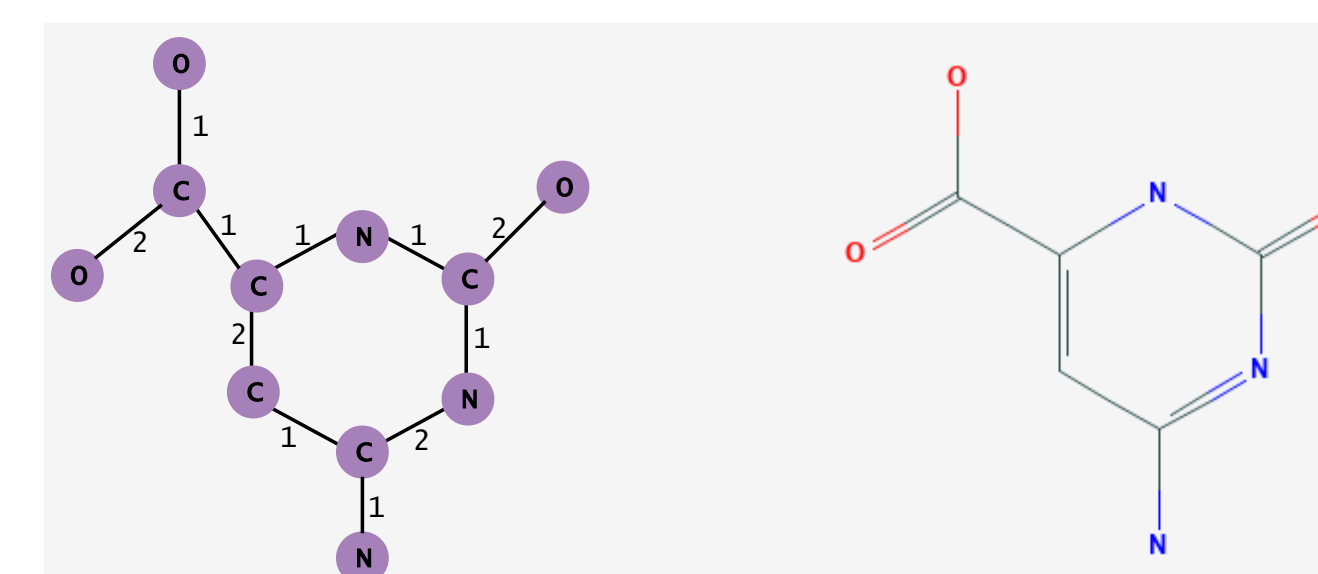


Fig 6. Chemical compound and its graph structure in IAM benchmark dataset

## Results

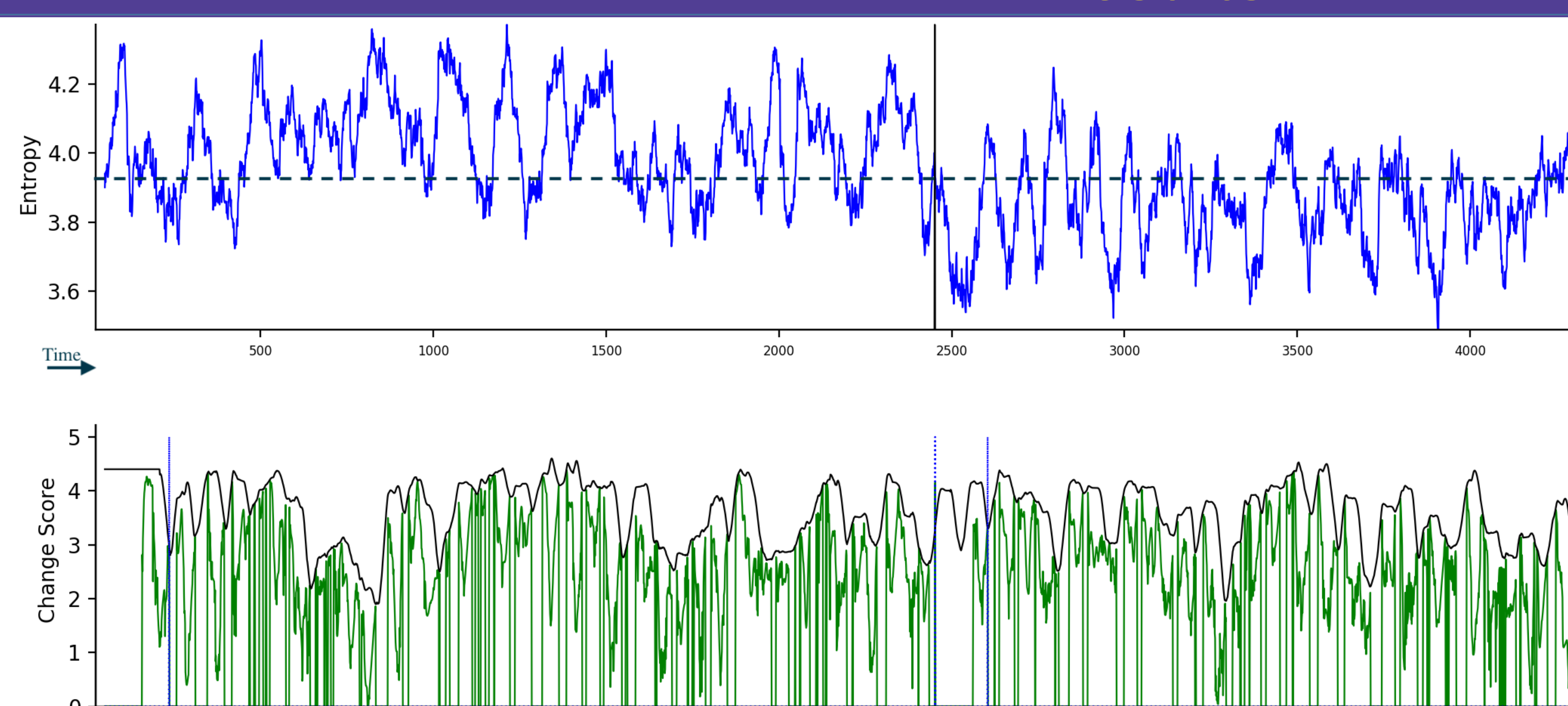


Fig 7. Entropy and change score on mutagenicity dataset  
X-axis represents time. The vertical black lines in the entropy plot are the real drift point. The black plot on the change score plot is the threshold and vertical dotted blue lines are the detected drift points.

Methods	DDR		DoD		FA1000	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
<b>DBLP Citation Network</b>						
DSDD	1.0	0	19.24	19.16	1.14	0.24
GEM	1.0	0	339	88.9	10.5	2.15
Zambon et. al.	1.0	0	437.5	-	0.08	0.08
<b>AIDS</b>						
DSDD	1.0	0	17.48	19.23	1.11	0.23
GEM	0	0	n/a	n/a	1.18	0.70
Zambon et. al.	1.0	0	62.5	-	1.31	0.45
<b>Mutagenicity</b>						
DSDD	1.0	0	22.8	20.63	0.60	0.18
GEM	0.46	0.50	124.12	161.0	1.37	0.42
Zambon et. al.	1.0	0	187.5	-	0.29	0.30
<b>Network DoS Attack</b>						
DSDD	1.0	0	21.16	21.24	0.81	0.25
GEM	1.0	0	4.92	1.63	11.05	2.98
Zambon et. al.	1.0	0	62.5	-	0.065	0.048

Table 2. Result obtained by DSDD and baseline methods on real-world datasets  
DDR (Drift Detection Rate), DoD (Delay of Detection), FA1000 (False Anomalies per thousand time step)

## Reference

- Song Liu, Makoto Yamada, Nigel Collier, and Masashi Sugiyama. 2013. Change point detection in time-series data by relative density-ratio estimation. Neural Networks 43 (2013), 72–83.
- Diane J Cook and Lawrence B Holder. 1993. Substructure discovery using minimum description length and background knowledge. Journal of Artificial Intelligence Research 1 (1993), 231–255.
- Yibo Yao and Lawrence B Holder. 2016. Detecting concept drift in classification over streaming graphs. In KDD Workshop on Mining and Learning with Graphs (MLG), San Francisco, CA. 2134–2142.
- Daniele Zambon, Cesare Alippi, and Lorenzo Livi. 2018. Concept drift and anomaly detection in graph streams. IEEE transactions on neural networks and learning systems 29, 99 (2018), 1–14..

## Step 3 – Change Detection

- Use direct density-ratio estimation approach called Relative Unconstrained Least-Squares Importance Fitting (RuLSIF) [1].
- The  $\alpha$ -relative PE divergence ( $\hat{PE}_\alpha$ ) gives the estimate of change.

$$\hat{PE}_\alpha = -\frac{\alpha}{2n} \sum_{i=1}^n \hat{g}(Y_i)^2 - \frac{1-\alpha}{2n} \sum_{j=1}^n \hat{g}(Y'_j)^2 + \frac{1}{n} \sum_{i=1}^n \hat{g}(Y'_i) - \frac{1}{2}$$

where  $\hat{g}(Y) = \sum_{l=1}^n \hat{\theta}_l K(Y, Y_l)$  and  $K(Y, Y_l) = \exp\left(-\frac{\|Y - Y_l\|^2}{2\sigma^2}\right)$

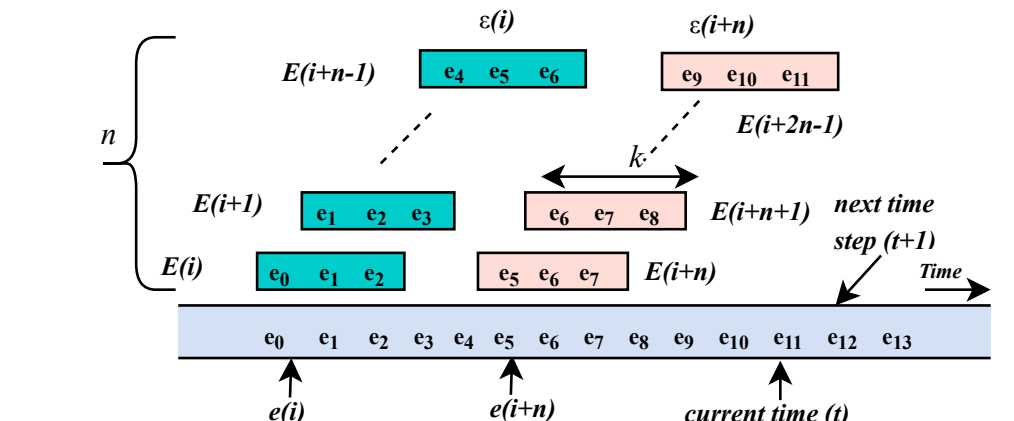


Fig 3. Change detection setup on entropy sequence

## Experiment

- Each experiment is run 50 times by randomizing the graphs in the streams keeping the drift point the same.
- Using synthetic graph streams, the effect of the window size  $|W|$  is studied.
- Using the best  $W$  ( $W = 50$ ), the performance of DSDD with GEM [3] and Zambon et al. [4] on 4 different real-world datasets is compared.
- For drift detection (step - 3),  $n = 50$ ,  $k = 10$ , and  $\alpha = 0.1$  are used.

## Conclusion

- Proposed a novel unsupervised algorithm for drift detection on graph streams called **Discriminative Subgraph-based Drift Detection (DSDD)**.
- Performed several experiments on synthetic as well as real-world data.
- It outperformed both baseline approaches in term of the DoD.
- Similar DDR and FA1000 with Zambon et. al.
- In conclusion, the unsupervised nature of DSDD makes it a more robust in a streaming scenario.

## Future Directions

- Investigate the scalability of our approach.
- Use our drift detection approach along with other learning models and investigate if the accuracy of the learning model can be improved by effectively detecting the drift.
- Test the performance against gradual drifts scenarios.

## Acknowledgements

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